University of Wisconsin - Madison

Supergravity Solutions in the Low-$\tan\beta$ λ_t Fixed Point Region *

V. Barger, M. S. Berger, P. Ohmann

Physics Department, University of Wisconsin, Madison, WI 53706, USA

Abstract

There has been much discussion in the literature about applying the radiative electroweak symmetry breaking (EWSB) requirement to GUT models with supergravity. We motivate and discuss the application of the EWSB requirement to the low $\tan\beta$ fixed-point region and describe the solutions we find.

1 Introduction

Improvements in LEP data over the past few years have generated significant excitement at the prospect of grand unification within the Minimal Supersymmetric Standard Model (MSSM) [1]. In addition to the gauge coupling unification suggested by LEP, Yukawa unification – in particular $\lambda_b(M_G) = \lambda_\tau(M_G)$ [2] – has been extensively studied, both at the one-loop and two-loop levels[3]. Such a constraint places significant restrictions on the allowed parameter space, especially that of m_t and $\tan\beta$. For values of $m_b(m_b)$ within the range 4.25 ± 0.10 GeV [4], the resulting allowed parameter space lies almost exclusively within the fixed - point region, as defined by $\lambda_i^G \gtrsim 1$ for $i = t, b$, and/or τ [3],[5]–[11].

If one makes only the additional assumption that $m_t(m_t) \lesssim 175$ GeV (consistent with the recently released CDF measurement $m_t^{pole} = 174 \pm 10 \,{}^{+13}_{-12}$ GeV[12] which corresponds to a running mass $m_t(m_t) \simeq 166 \pm 10 \pm 13$ GeV, then one is restricted to two very narrow regions in the m_t, $\tan\beta$ plane. One of these regions, the low $\tan\beta$ fixed-point region, has been the focus of our recent renormalization group analysis with supersymmetric grand unification[5], and we therefore examine whether these solutions satisfy the additional constraint imposed by Radiative Electroweak Symmetry Breaking (EWSB).

---

*Talk presented by P. Ohmann at the *Second IFT Workshop: Yukawa Couplings and the Origins of Mass*, Gainesville, Florida, February 11–13, 1994.

2   Fixed Points and $\lambda_b = \lambda_\tau$ Unification

Fixed-points arise naturally from imposing $\lambda_b = \lambda_\tau$ unification at the GUT scale along with the typically allowed range for the bottom quark mass 4.25 ± 0.10 GeV. Figure 1 shows the allowed parameter space for $\lambda_b = \lambda_\tau$ unification, and Figure 2 shows contours of Yukawa couplings (at M_{GUT}).
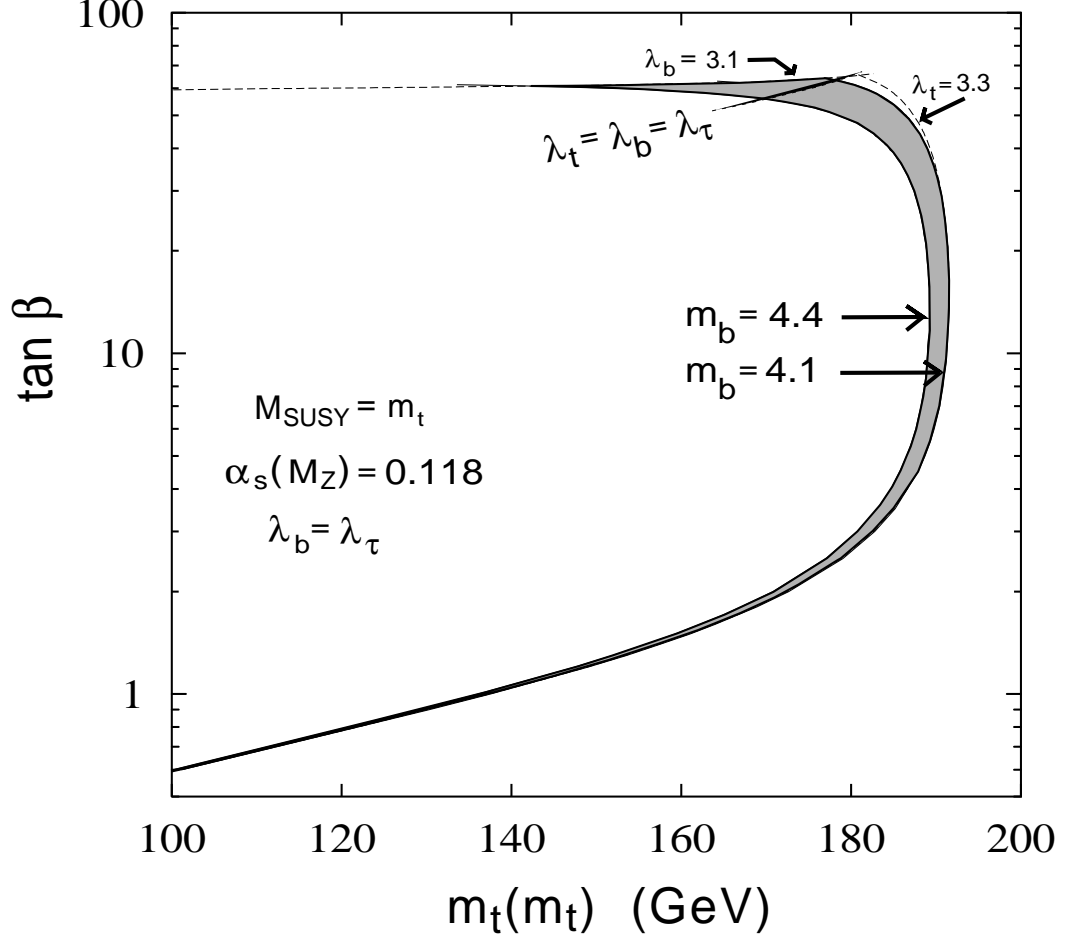

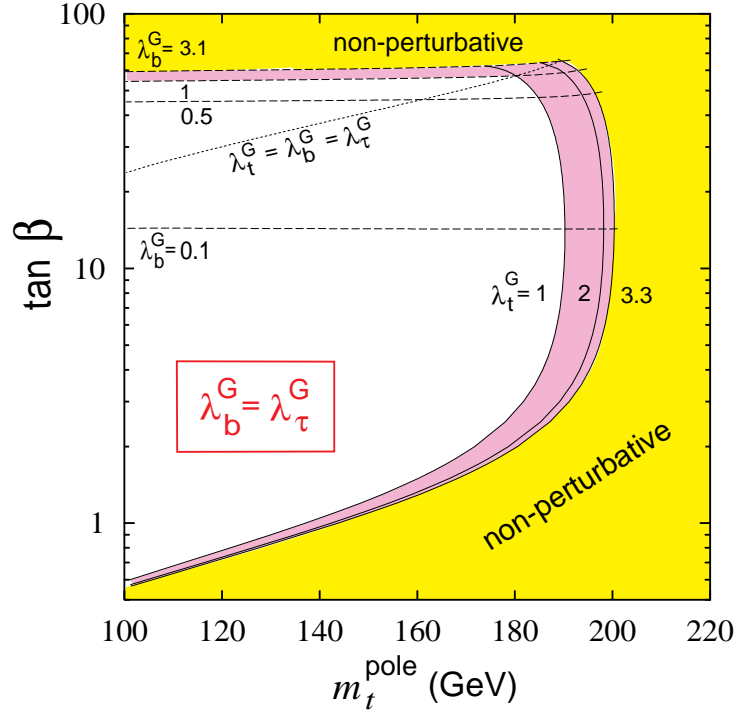Fig. 1. Contours of constant $m_b(m_b)$ in the $m_t(m_t)$, $\tan\beta$ plane (from Ref. [3]).

Fig. 2. The fixed-point regions are given by Yukawa couplings at the GUT scale being larger than about 1 ($\lambda_i^G \gtrsim 1$). Even larger values of the Yukawa couplings results in a breakdown of perturbation theory.

Note that Figure 1 is a subset of Figure 2 (allowing for the small $\sim$ 5-10 GeV difference between $m_t(m_t)$ and m_t^{pole}); in fact, imposing this m_b mass constraint ensures the fixed-point nature of the solutions. Figure 3 shows the typical evolution of λ_t for these solutions.
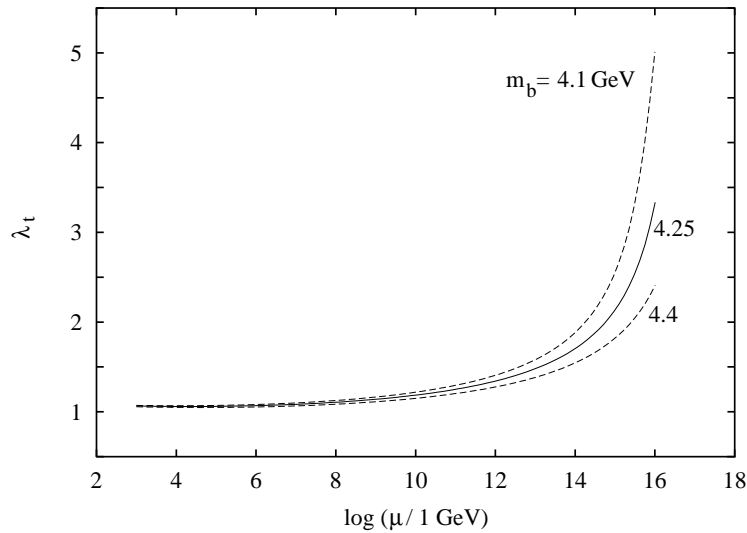

Fig. 3. If λ_t is large at M_G, then the renormalization group equation causes $\lambda_t(Q)$ to evolve rapidly towards an infrared fixed point as $Q \to m_t$ (from Ref. [3]).

3

As described by Figures 1 and 2, the allowed m_t-$\tan\beta$ parameter space can be divided into three distinct regions:

1) $\tan\beta \lesssim 2$ (λ_t fixed point)

2) $\tan\beta \gtrsim 50$ ($\lambda_b = \lambda_\tau$ fixed point)

3) $2 \lesssim \tan\beta \lesssim 50$ (λ_t fixed point)

It should be noted that threshold corrections, if large, may either enforce or mitigate the fixed-point nature of some of the solutions [8]–[10],[13]–[17]. If $m_t \lesssim 175$ GeV, then only the first two regions remain. While both solution sets may still be viable, there are criteria which seem to favor the first region: namely, the large $\tan\beta$ region typically results in large threshold corrections and in large enhancements to flavor changing neutral currents in processes like $b \to s\gamma$ and $B\overline{B}$ mixing and to proton decay[18]–[19]. However, it is possible these may be successfully eliminated by assuming certain symmetries[9].

One of the most important aspects of the large top mass is that it makes possible an understanding of the radiative breaking of the electroweak symmetry; the large top quark Yukawa drives a Higgs mass-squared negative. However, when both the top and bottom quark Yukawas are the same size at the GUT scale, one must rely on the difference in their hypercharges to effect the symmetry breakdown [20].

The low $\tan\beta$ λ_t fixed-point region can be well described by the following relation between the top quark mass and $\tan\beta$.

$$\lambda_t(m_t) = \frac{\sqrt{2}m_t(m_t)}{v\sin\beta} \approx 1.1 \;\Rightarrow\; m_t(m_t) \approx \frac{v}{\sqrt{2}}\sin\beta = (192\text{GeV})\sin\beta \quad (1)$$

Converting this relation to the top quark pole mass yields[3, 5]

$$m_t^{\text{pole}} \approx (200\text{GeV})\sin\beta \;. \tag{2}$$

3 Electroweak Symmetry Breaking

There has been much discussion in the literature about imposing a Radiative Electroweak Symmetry Breaking (EWSB) constraint on GUT models [21]–[33]; in particular we address this issue with regard to the low $\tan\beta$ λ_t fixed-point region.

The EWSB constraint is enforced by minimizing the effective Higgs potential; at tree-level this is given by:

$$\begin{aligned} V_0 =\; & (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 + m_3^2(\epsilon_{ij}H_1{}^i H_2{}^j + \text{h.c.}) \\ & + \frac{1}{8}(g^2 + g'^2)\left[|H_1|^2 - |H_2|^2\right]^2 + \frac{1}{2}g^2|H_1^{i*}H_2^i|^2 \;, \end{aligned} \tag{3}$$

where $m_{H_1}^2$, $m_{H_2}^2$, and $m_3^2 = B\mu$ are soft-supersymmetry breaking parameters, ϵ_{ij} is the antisymmetric tensor, and H_1 and H_2 are complex doublets given by

$$H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(\psi_1 + v_1 + i\phi_1) \\ H_1^- \end{pmatrix},$$

4

$$H_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(\psi_2 + v_2 + i\phi_2) \end{pmatrix}.$$

(4)

Minimizing this potential with respect to the two real components of the neutral Higgs fields ψ_1 and ψ_2 yields the tree-level EWSB minimization conditions:

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 ,$$

(5)

$$-B\mu = \frac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)\sin 2\beta .$$

(6)

The masses in these equations are running masses that depend on the scale Q in the RGEs that describe their evolution. Hence the solutions obtained are functions of the scale Q. Equations (5) and (6) are particularly convenient since the gauge couplings dependence (the D-terms in the language of supersymmetry) is isolated in Eq. (5). In addition, these minimization equations are readily solvable (even at the one-loop level) with the ambidextrous approach, which we describe in the next section.

The minimization equations also clearly show the fine-tuning problem that may be present in the radiative breaking of the electroweak symmetry. For large values of $|\mu|$, there must be a cancellation between large terms on the right hand side of equation (5) to obtain the correct experimentally measured M_Z (or equivalently the electroweak scale). For $\tan\beta$ near one, a cancellation of large terms must occur.

A heavy top quark produces large corrections to the Higgs potential of the MSSM[34]. Gamberini, Ridolfi, and Zwirner showed[23] that the tree-level Higgs potential is inadequate for the purpose of analyzing radiative breaking of the electroweak symmetry because the tree-level Higgs vacuum expectation values v_1 and v_2 are very sensitive to the scale at which the renormalization group equations are evaluated. The one-loop contribution to the effective potential is given by

$$\Delta V_1 = \frac{1}{64\pi^2}\text{Str}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{Q^2} - \frac{3}{2}\right)\right] ,$$

(7)

where ΔV_1 is given in the dimensional reduction ($\overline{DR}$) renormalization scheme[35]. The supertrace is defined as $\text{Str} f(\mathcal{M}^2) = \sum_i C_i(-1)^{2s_i}(2s_i+1)f(m_i^2)$ where C_i is the color degrees of freedom and s_i is the spin of the ith particle.

The one-loop corrections to the Higgs potential effectively moderates this sensitivity to the scale Q. The one-loop corrections are conveniently calculated using the tadpole method[32],[36],[37]. The one-loop corrected minimization conditions can then be used to generate a complete supersymmetric particle spectrum which satisfies EWSB. Including only the leading contribution coming from the top quark loop (and neglecting the D-term contributions to the squark

5

masses) one obtains the expressions

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 - \frac{3g^2 m_t^2}{32\pi^2 M_W^2 \cos 2\beta}\left[2f(m_t^2) - f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)\right.$$
$$\left. + \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left((\mu\cot\beta)^2 - A_t^2\right)\right] ,$$

(8)

$$-B\mu = \frac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)\sin 2\beta - \frac{3g^2 m_t^2 \cot\beta}{32\pi^2 M_W^2}\left[2f(m_t^2) - f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)\right.$$
$$\left. - \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}(A_t + \mu\cot\beta)(A_t + \mu\tan\beta)\right] ,$$

(9)

where

$$f(m^2) = m^2\left(\ln\frac{m^2}{Q^2} - 1\right) .$$

(10)

The extra one-loop contribution included above renders the solution less sensitive to the scale Q[26]–[29], [32],[27], as can be shown explicitly by examining the relevant renormalization group equations for the parameters that enter into the minimization conditions. The complete expressions for the one-loop contributions can be found in Ref. [32]. The fine-tuning problem is alleviated somewhat, but not entirely, by the inclusion of one-loop corrections to the Higgs potential. As our naturalness criterion we require

$$|\mu(m_t)| < 500 \text{ GeV} .$$

(11)

4   Ambidextrous Approach

Other RGE studies of the supersymmetric particle spectrum have evolved from inputs at the GUT scale (the top-down method[38]) or from inputs at the electroweak scale (the bottom-up approach[28]). The ambidextrous approach [39] incorporates some boundary conditions at both electroweak and GUT scales. We specify m_t and $\tan\beta$ at the electroweak scale (along with M_Z and M_W) and the common gaugino mass $m_{\frac{1}{2}}$, scalar mass m_0, and trilinear coupling A^G at the GUT scale. The soft supersymmetry breaking parameters are evolved from the GUT scale to the electroweak scale and then $\mu(M_Z)$ and $B(M_Z)$ (or $\mu(m_t)$ and $B(m_t)$) are determined by the one-loop minimization equations.

This strategy is effective because the RGEs for the soft-supersymmetry breaking parameters do not depend on μ and B. This method has two powerful advantages: First, any point in the m_t – $\tan\beta$ plane can be readily investigated in specific supergravity models since m_t and $\tan\beta$ are taken as inputs. Second, the minimization equations

are easy to solve in the ambidextrous approach: equation (8) can be solved iteratively for $\mu(M_Z)$ (to within a sign), and then equation (9) explicitly gives $B(M_Z)$. We stress the numerical simplicity: no derivatives need be calculated and no functions need to be numerically minimized.

5   Low $\tan\beta$ Fixed Point Solutions

We now describe our numerical approach in more detail. Starting with our low-energy choices for m_t, $\tan\beta$, α_3, and m_b (and using the experimentally determined values for α_1, α_2 and m_τ[40]), we integrate the MSSM RGEs from m_t to M_G with M_G taken to be the scale Q at which $\alpha_1(Q) = \alpha_2(Q)$. We then specify $m_{\frac{1}{2}}$, m_0, and A at M_G, and integrate back down to m_t where we solve the full one-loop minimization equations (see Ref. [32]) for $\mu(m_t)$ and $B(m_t)$. We can then integrate the RGEs back to M_G to obtain $\mu(M_G)$ and $B(M_G)$.

In particular, we choose values of m_t, $\tan\beta$, α_3, and m_b representative of the low $\tan\beta$ λ_t fixed point region. In addition to requiring EWSB to be satisfied, we impose the following experimental bounds:

Table 1. Approximate experimental bounds.

Particle	Experimental Limit (GeV)
gluino	120
squark, slepton	45
chargino	45
neutralino	20
light higgs	60

Together with our naturalness criteria $|\mu(m_t)| < 500$ GeV, these bounds give the allowed region in the $m_0, m_{1/2}$ plane shown as the shaded areas in Fig. 4.
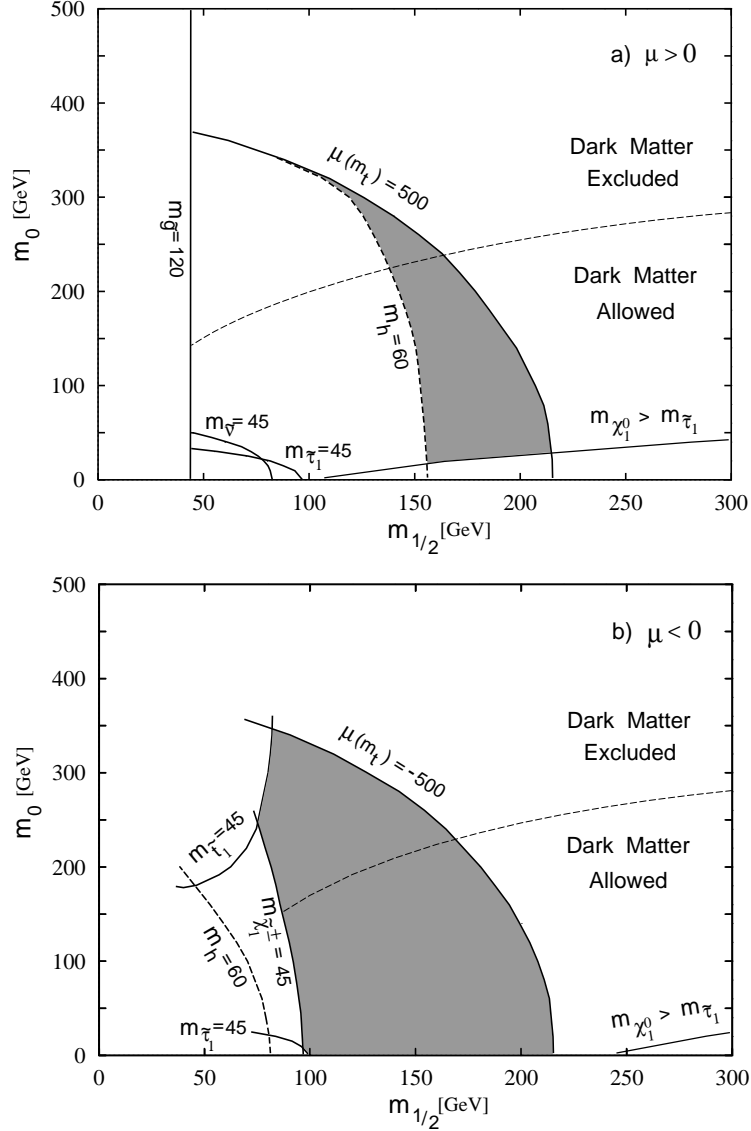
Fig. 4. Allowed regions of parameter space for $m_t(m_t) = 160$ GeV, $\tan\beta = 1.47$ (a low-$\tan\beta$ λ_t fixed-point solution) (from Ref. [32]).

Hence there are solutions in the low-$\tan\beta$ λ_t fixed-point region which satisfy EWSB constraints (as well as our naturalness criterion) at the one-loop level. Note that the $\mu < 0$ solutions have more allowed parameter space than do the $\mu > 0$ solutions. A few additional remarks are pertinent: the prediction for m_h in the low-$\tan\beta$ region is particularly sensitive to higher order corrections[16],[41],[42]. Hence the precise location of the $m_h = 60$ GeV contour is somewhat uncertain. Also, the dark matter line in Figure 4 should be regarded as semi-quantitative only since the contributions of s-channel poles that can enhance the annihilation rate have been neglected [43].

6   Conclusions

Given only two reasonable assumptions

- unification of couplings and $\lambda_b(M_G) = \lambda_\tau(M_G)$ at the GUT scale.

- $m_b(m_b) = 4.25 \pm 0.10$ GeV

we are restricted to the fixed-point region of $m_t - \tan\beta$ parameter space. With only one additional assumption, $m_t(m_t) \lesssim 175$ GeV, we are restricted to either

1) $\tan\beta \lesssim 2$ (λ_t fixed point), or

2) $\tan\beta \gtrsim 50$ ($\lambda_b = \lambda_\tau$ fixed point). The small $\tan\beta$ solution is favored by proton decay and flavor changing neutral current constraints. We investigated the additional constraint imposed by radiative electroweak symmetry breaking upon this first region, and found solutions which are both experimentally viable and meet the naturalness criterion $|\mu(M_Z)| \simeq |\mu(m_t)| < 500$ GeV.

7   Acknowledgements

References

[1] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. **B260** (1991) 447; J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. **B260** 131 (1991); P. Langacker and M. Luo, Phys. Rev. **D44** 817 (1991).

[2] M. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. **B128**, 506 (1977).

[3] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. **D47**, 1093 (1993).

[4] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[5] V. Barger, M. S. Berger, P. Ohmann, and R. J. N. Phillips, Phys. Lett. **B314**, 351 (1993), V. Barger, M. S. Berger, T. Han, and M. Zralek, Phys. Rev. Lett. **68**, 3394 (1992).

[6] B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981); C. T. Hill, Phys. Rev. **D24**, 691 (1981).

[7] C. D. Froggatt, I. G. Knowles, and R. G. Moorhouse, Phys. Lett. **B249**, 273 (1990); **B298**, 356 (1993).

[8] M. Carena, S. Pokorski, and C. E. M. Wagner, Nucl. Phys. **B406**, 59 (1993); W. Bardeen, M. Carena, S. Pokorski, and C. E. M. Wagner, Phys. Lett. **B320**, 110 (1994).

[9] L. J. Hall, R. Rattazzi, and U. Sarid, Lawrence Berkeley Lab preprint LBL-33997 (1993).

[10] P. Langacker and N. Polonsky, Phys. Rev. **D49**, 1454 (1994); N. Polonsky, Talk presented at the XVI Kazimierz Meeting on Elementary Particle Physics, Penn U. preprint UPR-0588-T, Kazimierz, Poland (1993), hep-ph 9310292.

[11] B. C. Allanach and S. F. King, University of Southampton preprint SHEP-93/94-15 (1994), hep-ph 9403212.

[12] CDF Collaboration, Fermilab preprint, April 1994.

[13] R. Barbieri and L. J. Hall, Phys. Rev. Lett. **68**, 752 (1992).

[14] L. J. Hall and U. Sarid, Phys. Rev. Lett. **70**, 2673 (1993).

[15] R. Hempfling, DESY preprint DESY-93-092 (1993).

[16] P. Langacker and N. Polonsky, Univ. of Pennsylvania preprint UPR-0594T (1994).

[17] B. D. Wright, University of Wisconsin preprint MAD/PH/812 (1994).

[18] R. Arnowitt and P. Nath, Phys. Rev. **70**, 3696 (1993).

[19] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. **69**, 1014 (1992); Nucl. Phys. **B402**, 46 (1993).

[20] M. Carena, M. Olechowski, S. Pokorski, and C. E. M. Wagner, Max Planck Institute preprint MPI-Ph/93-103(1994), hep-ph 9402253.

[21] For a review see P. Nath, R. Arnowitt, and A. H. Chamseddine, "Applied $N = 1$ Supergravity," World Scientific (1984).

[22] L. E. Ibañez and G. G. Ross, Phys. Lett. **B110**, 215 (1982); L. Alvarez-Gaumé, M. Claudson and M. B. Wise, Nucl. Phys. **207**, 96 (1982); J. Ellis, D. V. Nanopoulos, and K. Tamvakis, Phys. Lett. **B121**, 123 (1983); L. E. Ibañez, Nucl. Phys. **218**, 514 (1983); L. Alvarez-Gaumé, J. Polchinski and M. B. Wise, Nucl. Phys. **221**, 495 (1983); J. Ellis, J. S. Hagelin, D. V. Nanopoulos, and K. Tamvakis, Phys. Lett. **B125**, 275 (1983); L. E. Ibañez and C. Lopez, Phys. Lett. **B126**, 54 (1983); Nucl. Phys. **B233**, 511 (1984); C. Kounnas, A. B. Lahanas, D. V. Nanopoulos, and M. Quiros, Nucl. Phys. **B236**, 438 (1984); U. Ellwanger, Nucl. Phys. **B238**, 665 (1984); L. E. Ibañez, C. Lopez, and C. Muñoz, Nucl. Phys. **B256**, 218 (1985); A. Bouquet, J. Kaplan, and C. A. Savoy, Nucl. Phys. **B262**, 299 (1985); H. J. Kappen, Phys. Rev. **D35**, 3923 (1987); M. Drees, Phys. Rev. **D38**, 718 (1988); H. J. Kappen, Phys. Rev. **D38**, 721 (1988); B. Gato, Z. Phys. **C37**, 407 (1988); G. F. Giudice and G. Ridolfi, Z. Phys. **C41**, 447 (1988); S. P. Li and H. Navelet, Nucl. Phys. **B319**, 239 (1989).

[23] G. Gamberini, G. Ridolfi, and F. Zwirner, Nucl. Phys. **B331**, 331 (1990).

[24] R. Arnowitt and P. Nath, Phys. Rev. **69**, 725 (1992); Phys. Lett. **B289**, 368 (1992).

[25] G. G. Ross and R. G. Roberts, Nucl. Phys. **B377**, 571 (1992).

[26] S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, Nucl. Phys. **B398**, 3 (1993).

[27] B. de Carlos and J. A. Casas, Phys. Lett. **B309**,320 (1993); B. de Carlos, CERN preprint CERN-TH-7024-93, Talk given at the *XVI International Warsaw Meeting on Elementary Particle Physics* (1993), hep-ph 9310232.

[28] M. Olechowski and S. Pokorski, Nucl. Phys. **B404**, 590 (1993).

[29] P. Ramond, Institute for Fundamental Theory Preprint UFIFT-HEP-93-13 (1993), hep-ph 9306311; D. J. Castaño, E. J. Piard, and P. Ramond, Institute for Fundamental Theory Preprint UFIFT-HEP-93-18 (1993), hep-ph 9308335.

[30] W. de Boer, R. Ehret, and D. I. Kazakov, Inst. für Experimentelle Kernphysik preprint IEKP-KA/93-13, Contribution to the International Symposium on Lepton Photon Interactions, Ithaca, NY (1993), hep-ph 9308238.

[31] M. Carena, M. Olechowski, S. Pokorski, and C. E. M. Wagner, CERN preprint CERN-TH.7060/93 (1993), hep-ph 9311222.

[32] V. Barger, M. S. Berger, and P. Ohmann, University of Wisconsin preprint MAD/PH/801 (1993), hep-ph 9311269.

[33] G. Kane, C. Kolda, L. Roszkowski, and J. D. Wells, University of Michigan preprint UM-TH-93-24 (1993), hep-ph 9312272.

[34] M. S. Berger, Phys. Rev. **D41**, 225 (1990).

[35] W. Siegel, Phys. Lett. **B84**, 193 (1979); D. M. Capper D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. **B167**, 479, (1980).

[36] S. Weinberg, Phys. Rev. **D7**, 2887 (1973); S. Y. Lee and A. M. Sciaccaluga, Nucl. Phys. **B96**, 435 (1975); K. T. Mahanthappa and M. A. Sher, Phys. Rev. **D22**, 1711 (1980).

[37] M. Sher, Phys. Rep. **C179**, 273 (1989).

[38] R. Hempfling, DESY preprint DESY 94-057.

[39] S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, K. Yuan, Phys. Lett. **B273**, 423 (1991).

[40] Particle Data Book, Phys. Rev. **D45** (1992).

[41] J. Kodaira, Y. Yasui, and K. Sasaki, Hiroshima preprint HUPD-9316 (1993).

[42] R. Hempfling and A. H. Hoang, DESY preprint DESY-93-162 (1993).

[43] M. Drees and M. M. Nojiri, Phys. Rev. **D47**, 376 (1993).